TRINITY ✠ COLLEGE

MATHEMATICS APPLICATIONS UNIT 3, 4

Section Two:
Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	10	10	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed **65% (98 Marks)**

This section has **ten (10)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(7 marks)**

(a) The first three terms, in order, of a geometric sequence are $900, 540$ and 324.

   (i) Deduce a rule for the nth term of this sequence. (2 marks)

   (ii) Calculate the 6th term of the sequence. (1 mark)

(b) The first three terms, in order, of an arithmetic sequence are $1.5, 4.2$ and 6.9.

   (i) The rule for the nth term of this sequence is $T_n = an + b$. Determine the values of a and b. (3 marks)

   (ii) Calculate the 316th term of the sequence. (1 mark)

See next page

Question 10 **(10 marks)**

A public relations company was tasked with determining whether a person's support for a sugary drinks tax could be associated with their interest in the news.

The company carried out a telephone survey, where people could respond to two questions as shown in the following table:

Question	Choice of response
Are you interested in the news?	Yes or No
Do you support a sugary drinks tax?	Yes, No or Undecided

The responses to the telephone survey are summarised in this table:

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes	556	367	233
	No	196	136	146

(a) Calculate the number of people who

(i) answered yes to being interested in the news. (1 mark)

(ii) responded to the survey. (1 mark)

(b) If there was no association between interest in the news and support for a sugary drinks tax, should the company expect a larger percentage of those who are interested in the news to support a sugary drinks tax compared to those who are not interested in the news? Explain your answer. (2 marks)

(c) Complete the two-way table below to show the associated **row** percentages for the previous table, rounding percentages to the nearest whole number. (3 marks)

		Support for a sugary drinks tax		
		Yes	No	Undecided
Interest in news	Yes			
	No			

(d) What percentage of those who are not interested in the news do not support a sugary drinks tax? (1 mark)

(e) In the context of the task they were given, how should the public relations company interpret the responses to their survey? (2 marks)

Question 11   **(8 marks)**

A sequence of four connected graphs is shown below.

G_1 G_2 G_3 G_4
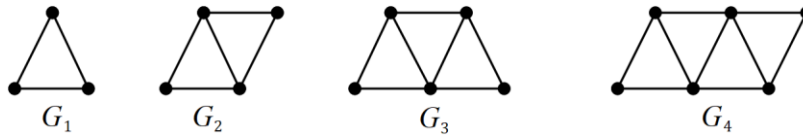
(a)   Complete the missing entries in the table below, where the vertex sum is the sum of the degrees of all the vertices in a graph.   **(2 marks)**

Graph (n)	1	2	3	4
Vertices (V)	3	4		
Vertex sum (S)	6	10		

Assume the sequence of graphs continues indefinitely.

(b)   A graph in the sequence has 11 faces. How many vertices does it have?   **(2 marks)**

(c)   Deduce the nth term rule for S_n, the vertex sum of graph n.   **(2 marks)**

(d)   A graph in the sequence has a vertex sum of 350. Determine the number of vertices this graph has.   **(2 marks)**

Question 12   **(9 marks)**

Data from the Australian Bureau of Statistics showing the quarterly beer production (in megalitres) over 23 quarters (December 2004 to June 2010) reveal a seasonal cycle with a period of 4 quarters. The 23 quarters are used to calculate 4-point centred moving averages.

(a)   What is the purpose of calculating moving averages for such data.   **(2 marks)**

(b)   Using D (the deseasonalised data) and t (where $t = 1$ for December 2004, $t = 2$ for March 2005, etc.), the equation of the regression line that models the trend component is

$$D = 424.24 - 0.0077t$$

Describe what the slope of this line indicates about beer production.   **(2 marks)**

(c)   The seasonal proportions for the June quarters are

$$0.96, 0.90, 0.90, 0.92, 0.93$$

     (i)   What do these seasonal proportions indicate about beer production in the June quarter?   **(1 mark)**

     (ii)   Calculate the seasonal index for June.   **(1 mark)**

(d)   Using an appropriate model, predict the number of megalitres of beer produced for the June quarter of 2013. Comment on your prediction.   **(3 marks)**

Question 13 **(15 marks)**

A tomato grower added varying amounts of a liquid fertiliser (x ml) to the irrigation systems of twelve greenhouses and observed the resulting yield of tomatoes per plant (y kg). A sample of the data recorded is shown in the table and scatterplot below.

x (mL)	6	9	10	12	14	15	17	18	20	21	24	26
y (kg)	17	21	19	23	24	24	25	22	23	26	25	28
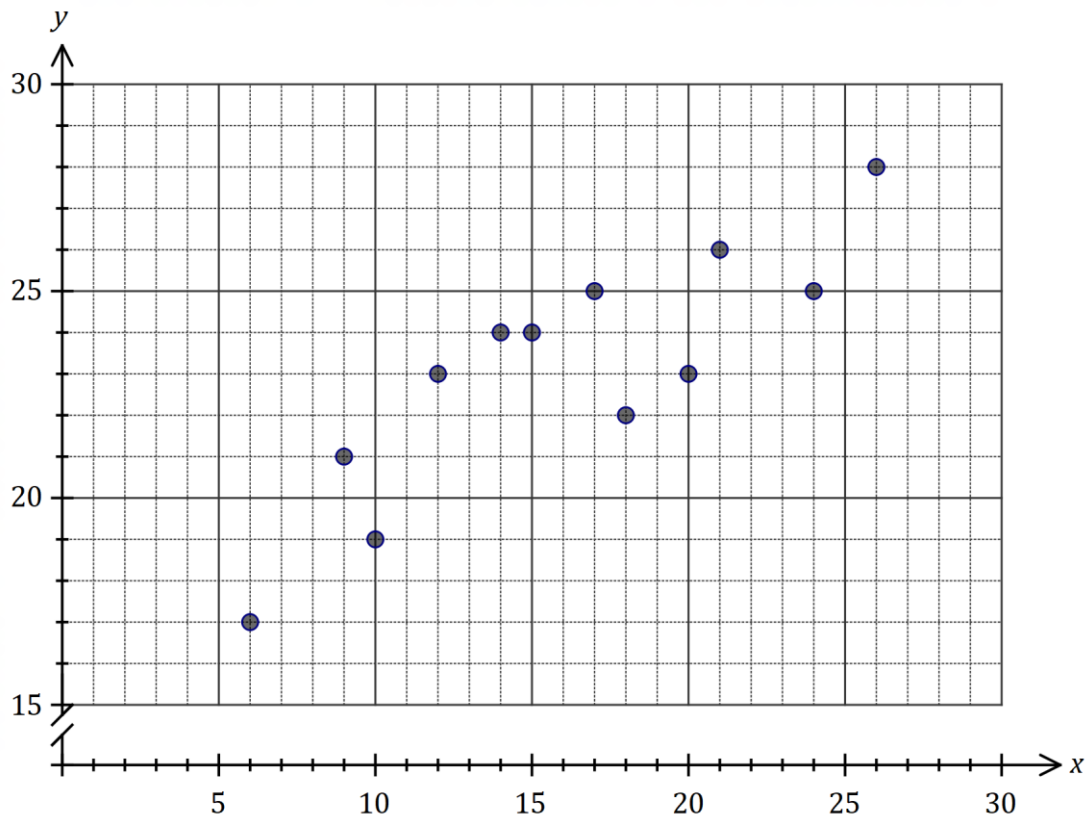
(a)   Name the response variable. (1 mark)

(b)   For this data, calculate

(i)   the correlation coefficient. (1 mark)

(ii)   the values of a and b in the equation of the least-squares line $y = ax + b$.
(2 marks)

See next page

(c)   What percentage of the variation in the yield per plant can be explained by the variation in
        the amount of liquid fertiliser added?   (1 mark)

(d)   If the amount of liquid fertiliser added to the irrigation system in a greenhouse was
        increased by one millilitre, what increase in the yield of tomatoes per plant can be
        expected? Explain your answer.   (2 marks)

(e)   If no liquid fertiliser was used, what yield of tomatoes per plant does the linear model
        predict?   (1 mark)

(f)   Use the equation of the least-squares line to calculate the value of y when $x = 5$ and
        when $x = 25$.   (2 marks)

(g)   Use your answers to part (f) to draw the least-squares line on the scatterplot.   (2 marks)

(h)   Estimate the yield of tomatoes per plant when 8 ml of liquid fertiliser is added to the
        irrigation system and comment on the reliability of this value.   (3 marks)

Question 14   **(10 marks)**

The roads in a suburb are represented in the graph below, where the number on each edge is the length, in km, of the road.
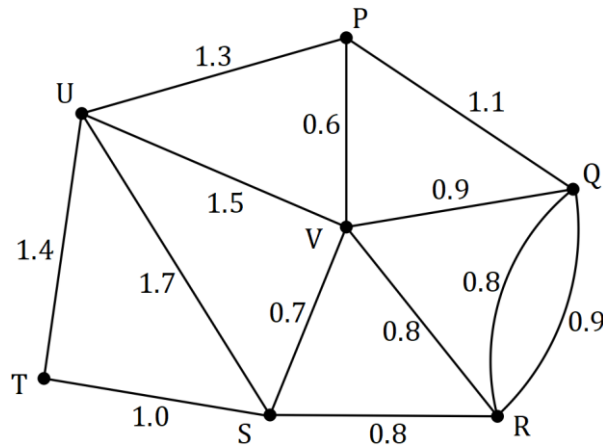
(a)   The network contains a semi-Eulerian trail that ends at P.

    (i)   Explain what semi-Eulerian means.   **(2 marks)**

    (ii)   At which vertex must the trail start?   **(1 mark)**

    (iii)   Determine the number of edges in the trail.   **(1 mark)**

    (iv)   How many times does the trail pass through vertex Q?   **(1 mark)**

(b)   A worker needs to leave V, travel along each road once to inspect its surface and then return to V. Determine the minimum distance the worker must travel.   (3 marks)

(c)   Another worker needs to walk along each road twice, once on each side of the road, as they deliver advertising pamphlets to houses. Determine, with justification, the minimum distance this worker must travel if they start and finish at S and the width of the roads is ignored.   (2 marks)

Question 15 (12 marks)

Measurements of temperature for Perth were recorded over a period of three years. Each measurement is the mean maximum temperature over a three-month season.

Season	Time period (t)	Temperature (T) ($°C$)	Moving average	Cycle mean	Seasonal proportion	Deseasonalise data (D)
Summer 2015	1	31.4			C	25.4
Autumn 2015	2	25.4		25.59	0.99	24.8
Winter 2015	3	19.7	25.6		0.77	25.7
Spring 2015	4	25.9	25.5		1.01	26.6
Summer 2016	5	31.2	25.3		1.29	25.2
Autumn 2016	6	25.1	24.7	B	1.04	24.6
Winter 2016	7	18.0	24.1		0.74	23.5
Spring 2016	8	22.7	24.0		0.93	23.2
Summer 2017	9	29.8	24.3		1.20	24.1
Autumn 2017	10	25.9	24.7	24.89	1.04	25.3
Winter 2017	11	19.5	24.9		0.78	25.5
Spring 2017	12	24.4			0.98	25.0
Summer 2018	13	A				24.0

(a) Using a 4-point centred moving average, calculate values for A, B and C in the table above. (3 marks)

See next page

(b)   In 2014, one season had a Seasonal proportion of 0.96. Which season do you think it
          was? Explain your answer.   (2 marks)

(c)   The trend for this data can be modelled by the regression line

$$D = 25.45 - 0.09t$$

          A newspaper article has stated that this provides evidence against global warming.

          (i)   What does the slope of the regression line indicate about the change in
                   temperature over time?   (2 marks)

          (ii)   Comment on the validity of the newspaper article's claim.   (2 marks)

          (iii)   A predicted mean temperature for Winter is $17.8°$. Determine which year this
                   prediction is from.   (3 marks)

Question 16   **(9 marks)**

The temperature, T °C, of an industrial oven n minutes after it is turned on can be modelled by

$$T_{n+1} = 0.85T_n + 28.5, \qquad T_0 = 18$$

(a) Use the recurrence relation to complete the table of values below, rounding the temperature to the nearest °C.   (2 marks)

n	0	5	10	15	20
T_n					

(b) Sketch a graph of the temperature of the oven for the first 45 minutes on the axes below. Make sure you add a suitable scale to the vertical axis.   (4 marks)
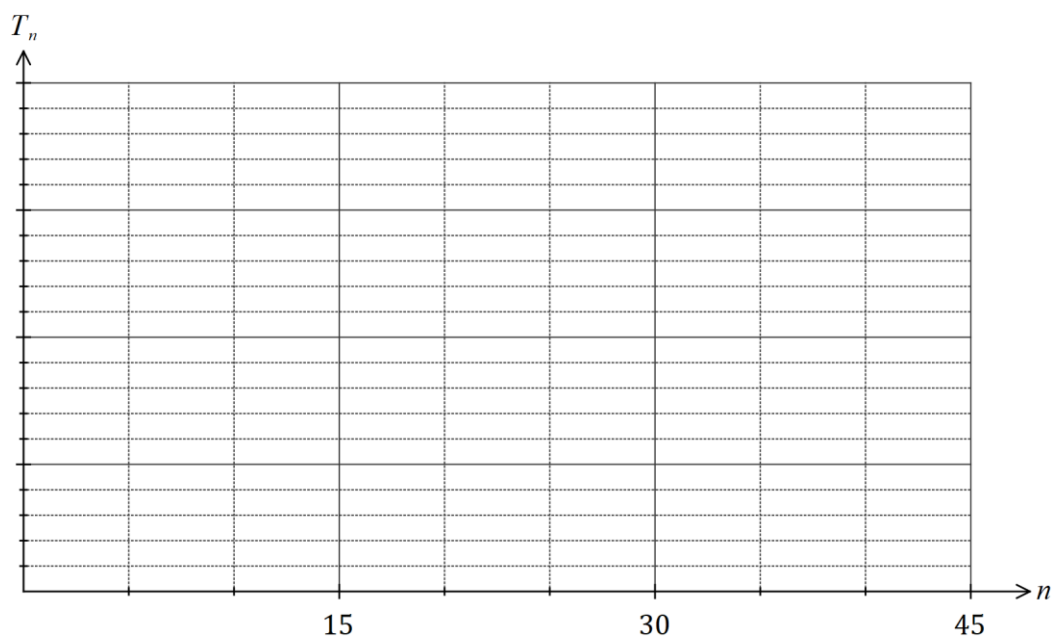

(c) The manufacturer claims that the oven will reach within 5 °C of its maximum temperature within 20 minutes of being turned on. Comment on this claim.   (3 marks)

See next page

Question 17   **(8 marks)**

A linear model was fitted to a set of data, resulting in a correlation coefficient of $r = -0.92$ and a least-squares line with equation $\hat{y} = 65.85 - 1.25x$. A residual plot for the linear model is shown below.
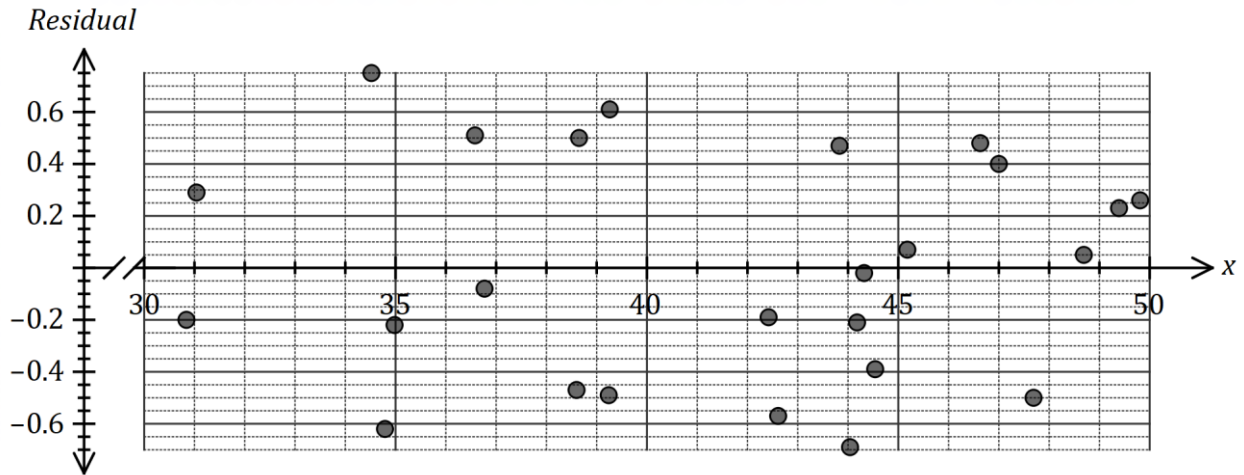
Residual

(a)   Calculate, and add to the plot above, the residual for the point $x = 33$, $y = 24.1$.

(3 marks)

(b)   Use the residual plot to comment on the appropriateness of fitting a linear model to the data.   (2 marks)

(c)   Determine the y-coordinate of the point with a residual of 0.4 on the above plot.

(3 marks)

See next page

Dara is visiting a city that has four museums: P, Q, R and S. The weights on the edges of the following graph represent the time, in minutes, that it takes to walk between the museums.
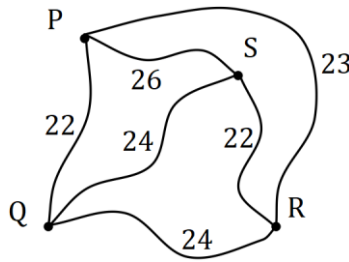
(a)   List, in the order visited, a set of vertices that form a Hamiltonian cycle in the graph.

(2 marks)

(b)   Determine the shortest time it would take to leave P, walk to the other museums and return to P.

(2 marks)

The time to walk from Dara's hotel, H, to museums P, Q, R and S is $26, 12, 28$ and 23 minutes respectively.

(c)   Add vertex H and this information to the graph above.   (2 marks)

(d)   Dara plans to leave her hotel at 8.30 am, visit all the museums and then return to her hotel at 2.30 pm the same day. Determine the maximum total time she can spend inside the museums if she walks between them and describe the route she should take.

(4 marks)

End of questions

Question number: _____

Supplementary page

Question number: _____

Supplementary page

Question number: _____